Dependence between components of multivariate Markov chains: Markov consistency and Markov Structures

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Modeling of evolution of dependence between processes occurring in financial markets is important. Typically, one can identify marginal statistical properties of individual processes, and then one is confronted with the task of modeling dependence between these individual processes so that the marginal properties are obeyed. We have been advocating, for some time now, to address this modeling problem via the theory of Markov consistency and Markov structures.

In this talk we shall examine the problem of existence and construction of a non-trivial multivariate Markov chain with components that are given Markov chains. In this regard we shall give sufficient and necessary conditions, in terms of relevant expectations, for a component of a multivariate Markov chain to be a Markov chain in the filtration of the entire chain - a property called strong Markov consistency, as well as in its own filtration - a property called weak Markov consistency. These characterization results are proved via analysis of the semi-martingale
structure of the chain.

Some financial applications will be indicated.